## Problem Set 12

Brexit

British scientist Duffy is eager to visit Jupiter's moon Europa and rides a submarine to search for alien life in its oceans below an icy crust. Europa's mass has been measured to be  $4.80 \times 10^{22}$  kg, its diameter is 3120 km, and it has no appreciable atmosphere. Assume that the layer of ice at the surface is not thick enough to exert substantial force on the water.

- a) If Duffy discovered alien life forms in the subsurface waters of Europa, what would Duffy call them? Will Duffy leave this moon?
- b) If the windows of the submarine each have an area of 625 cm<sup>2</sup> and can stand a maximum inward force of 8750 N per window, what is the greatest depth to which Duffy's submarine can safely dive?
- Gravity of a Ring
  - Consider the ring- shaped body of Fig. 12.1. A particle with mass m is placed a distance x from the center of the ring, along the line through the center of the ring and perpendicular to its plane.
    - a) Calculate the gravitational potential energy U of this system. Take the potential energy to be zero when the two objects are far apart.
    - b) Show that your answer to part (a) reduces to the expected result when x is much larger than the radius a of the ring.
    - c) Use  $F_x = -dU/dx$  to find the magnitude and direction of the force on the particle.
    - d) Show that your answer to part (c) reduces to the expected result when x is much larger than a.



- e) What are the values of U and  $F_x$  when x = 0? Explain why these results make sense.
- Nonuniform Planet

Planets are not uniform inside. Normally, they are densest at the center and have decreasing density outward toward the surface. Model a spherically symmetric planet, with the same radius as the earth, as having a density that decreases linearly with distance from the center. Let the density be  $15.0 \times 10^3$  kg/m<sup>3</sup> at the center and  $2.00 \times 10^3$  kg/m<sup>3</sup> at the surface. What is the acceleration due to gravity at the surface of this planet?

Marianas Trench

The deepest point known in any of the earth's oceans is in the Marianas Trench, 10.92 km deep.

- a) Assuming water is incompressible, what is the pressure at this depth? Use the density of seawater.
- b) The actual pressure is 1.16 × 10<sup>8</sup> Pa; your calculated value will be less because the density actually varies with depth. Using the compressibility of water and the actual pressure, find the density of the water at the bottom of the Marianas Trench. What is the percent change in the density of the water?
- Duffffffffy
  - An evolved Duffy, called Dufffffffy is 1.83 m tall.
    - a) Calculate the difference in blood pressure between the feet and top of the head of Dufffffffy.
    - b) Consider a cylindrical segment of a blood vessel 2.00 cm long and 1.50 mm in diameter. What additional outward force would such a vessel need to withstand in Dufffffffy's feet compared to a similar vessel in his head?

Star-Lord

*[Optional |maybe Challenging]* Duffy stole the *Milano* from Quill and cruised the ship towards a black hole accidently. Now Duffy is sitting inside the Milano and orbiting the black hole at 120 km from its center. The black hole is 5.00 times the mass of the sun and has a Schwarzschild radius of 15.0 km. Duffy is positioned inside the Milano such that one of his 0.030-kg ears is 6.00 cm farther from the black hole than the center of mass of the spacecraft and the other ear is 6.0 cm closer.

- a) Everyone knows that stealing is a crime in Marvel's universe. Now Duffy is facing his karma. What is the tension between Duffy's ears? Would Duffy find it difficult to keep from being torn apart by the gravitational forces? (Since Duffy's whole-body orbits with the same angular velocity, one ear is moving too slowly for the radius of its orbit and the other is moving too fast. Hence his head and tissues must exert forces on his ears to keep them in their orbits.)
- b) Is the center of gravity of Duffy's head at the same point as the center of mass?



